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Oceanographic Applications of Ranging to Artificial Satellites

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Oceanographic applications of ranging to artificial satellites

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The satellite-borne pulsed radar altimeter is in current use in providing practically continuous records of the height of certain satellites above the ocean surface, with a precision approaching 10–20 cm. This precision is largely wasted unless the geocentric position of the satellite is also monitored to a similar accuracy by ground-based laser tracking facilities. The combination will enable us to monitor the variations of the geocentric ocean surface in space and time within the range of the tracking centres. Much of the variation is due to the shape of the geoid, of great interest in itself, but detection of dynamic departures from the geoid, possible at decimetre accuracy, opens up new possibilities in ocean exploration. The greatest benefits will be to the determination of ocean (and Earth) tides, the open-sea behaviour of storm surges, and changes in topography due to varying patterns of circulation.

Introduction

The oceanographic applications which I shall discuss are those which depend on the precise determination of the geocentric level of the sea surface as a function of latitude, longitude and time. Such a determination is possible by laser ranging to an artificial satellite which bears a precision pulsed radar altimeter. Other applications of laser ranging to oceanography are possible but rather indirect. For example, study of the perturbations in the orbits of near-Earth satellites at tidal frequencies lead to estimates of the low-order harmonics of the solid Earth cum ocean tidal system, which practically define the energy loss in the oceanic tides. Such work is described in the paper by Cazenave, Daillet & Lambeck (1976) presented at this meeting. Precise measurements of the temporal variations of the Moon's orbital parameters and of the Earth's rotation, possible from lunar laser ranging, can also be linked to oceanic tidal friction (Lambeck 1975). Ranged altimetry promises to yield much more detailed information about dynamic and static properties of the ocean.

The potential uses of satellite altimetry in ocean surveying were discussed some years before a suitable instrument was actually constructed, notably by Greenwood *et al.* (1969). The impressive demonstration of the S-193 altimeter in Skylab (McGoogan, Leitao & Wells 1975) has now turned the instrumental concept into reality, but the coarseness of the tracking systems available for Skylab kept the absolute accuracy of geocentric altitudes in the ± 5 m class, despite the much better accuracy of the altimeter itself. Improvements in laser ranging technique promise absolute accuracy from the altimeter system recently launched in Geos-3 (previously known as Geos-C) in the order of ± 0.5 m (Leitao, Purdy & Brooks 1975). Continuing refinements, to be incorporated in the Seasat spacecraft (whose name aptly suggests its primary area of study), make 0.2 m geocentric accuracy a realistic hope for the near future. Such precision makes possible a serious discussion of problems in modern oceanography which may be solved by these new space techniques far more appropriately than by current surface-based methods.

Principles of measurement

I am totally unqualified to give a full exposition of the instrumental problems involved in altimetry and associated ranging, but a brief summary of the basic principles is necessary here, for the benefit of the uninitiated reader. Authoritative accounts may be found in the references cited in the preceding paragraph.

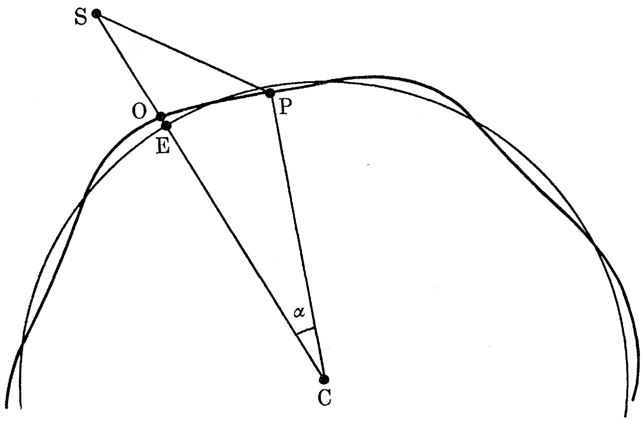

Figure 1. The basic geometry of ranged altimetry. The thin smooth curve represents the Reference Ellipsoid with centre C; the thick irregular curve, the actual surface of the Earth. P is a tracking station, and S the instantaneous position of a satellite above the ocean surface at O. PC and the angle α can be assumed known. Laser ranging measures SP, and hence SC. The altimeter measures SO, and hence the desired geocentric ocean level OC (or OE).

A high-powered, very short radar pulse (2 kW, 200 ns), is transmitted vertically downwards from the satellite, and the leading edge of its return from the sea surface is timed to about nanosecond accuracy, equivalent to about 0.3 m of altitude. Pulses are repeated 100 times/s, giving a horizontal separation along the sea surface of about 100 m, so oceanic features of order 1 km in horizontal scale or greater can be resolved to even higher precision by averaging over several consecutive, independent pulses. Systematic errors, due to variable transmissive properties of the atmosphere may be corrected in terms of known synoptic meteorological parameters. Short surface waves on the ocean impart a bias to the local mean sea level and also blur the edge of the return pulse. The blurring can be analysed instrumentally to give a measure of the mean height of the waves, which in turn allows a correction for the bias in sea level. In areas of steep geoidal features, some allowance has to be made for the fact that the nearest point of the ocean surface, which determines the leading edge of the return pulse, is not necessarily vertically below the satellite. When these, and other instrumental biases, have been removed, it is said that the satellite's altitude above the sea surface can be specified, effectively continuously along the orbital track, to an accuracy of order 0.2 m.

To obtain the desired geocentric sea surface level, the geocentric altitude of the satellite itself must also be known (see figure 1). Remotely from the tracking centres, the satellite's geocentric altitude can only be specified to the accuracy of a few metres. Here the precision of 0.2 m in altimetry is largely wasted. The measurements are still of great value to study of the shorter wavelength features of the geoid, but they are of little value to oceanography. Near a laser tracking station however, the distance of the satellite from the tracking station can be specified to an accuracy which easily matches, or can surpass that of the altimeter, as discussed elsewhere at this meeting. The geocentric position of the station itself may also be assumed known in the present context, or may be deduced after repeated measurements. (A correction for the local Earth tide is straightforward.) The geocentric altitude of the satellite then follows by simple geometry, provided we also have an estimate of its true angular distance from the tracking station. Atmospheric refraction prevents a direct measurement of this angle, but it may be derived from ranges from other tracking stations during the same orbit, and from precise knowledge of the mean orbital characteristics.

Figure 1 summarizes visually the geometrical relationships between the measurements discussed above.

Variations in sea level

(a) *Static features*

I shall now outline what is known about the vertical variations of the ocean surface of large horizontal scale, and emphasize those areas where our knowledge is insufficient but should be improved by ranged altimetry.

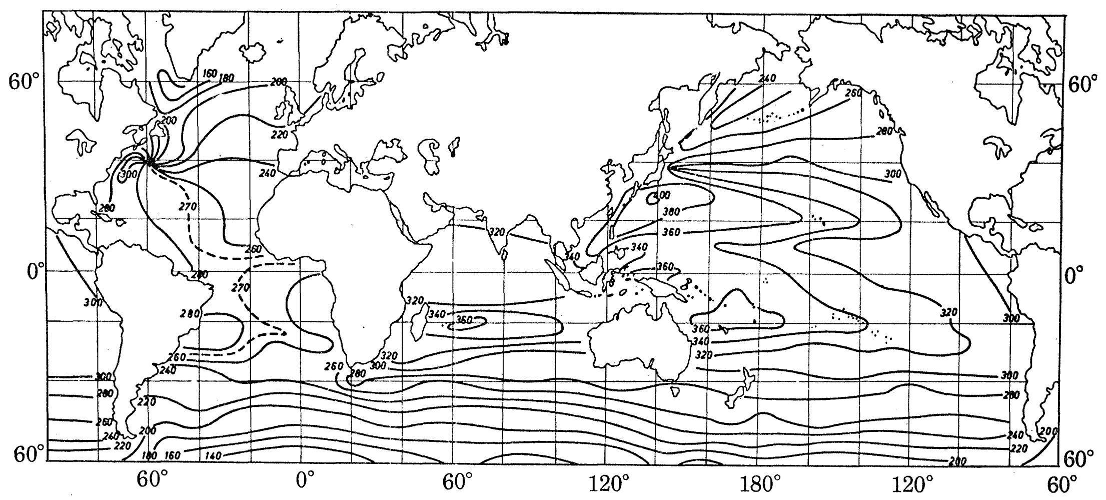

Figure 2. Contours of the mean ocean surface, relative to the Geoid, according to Lisitzin (1974). Units are centimetres.

In the language of coastal geodesy, the term 'mean sea level' is used to denote the altitude of the level of the sea, measured by a tide gauge relative to a nearby mark fixed to the land, after tides and random variations in time due to meteorological effects have been removed by averaging over a specified length of time such as one year. The altitude of the land-mark itself (which also varies tidally) is related to the altitudes of similar marks over the contiguous land mass, essentially by spirit-levelling. Thus, the mean sea levels round the coast of any land mass may be referred to a common, gravitationally level or equipotential surface. Many years ago, geodesists used to consider mean sea level as itself defining an equipotential surface, but fairly precise land levelling between tide gauges has since proved that this cannot be so. Differences of order $\frac{1}{2}$ m between mean sea levels at various coastal sites have been found, especially from one ocean to another, as from the Atlantic to the Pacific coasts of America, but also along a coastline of the same ocean, as in eastern Australia.

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Oceanographers understand quite well why mean sea level should differ from a gravitational level surface, although oceanographic calculations of differences in altitude often disagree with the results from land levelling. Setting aside small-scale effects particular to very shallow seas and strong currents, sea level between two points on the ocean will differ if:

(*a*) the integrated specific volumes of water from the sea surface to a deep reference level (e.g. 4000 m) differ, or

(*b*) a current flows transversely to the line joining the points, or

(*c*) the mean atmospheric pressures differ.

Since (*a*) and (*b*) are related by the geostrophic equation, the sea-level anomalies can be computed from available data for either (*a*) and (*c*) or for (*b*) and (*c*), with equivalent results. Figure 2 shows a map of sea level anomaly in the world oceans, as computed by Lisitzin (1974) from (*a*) and (*c*). Differences of more than 2.5 m can be found; there are steep gradients between the Atlantic and Pacific Oceans owing to the different sea densities, and across the Gulf Stream and other concentrated current systems.

However, it is a longstanding fact that maps similar to figure 2 fail to reproduce certain anomalies in sea level derived from land-levelling, notably in north–south directions. For example, a northerly rise of about 1.7 m is claimed for the 2500 km eastern Australian seaboard, while figure 2 shows a rise of barely 0.6 m. The intensive land levelling of the smaller area of Britain persistently suggests that mean sea level off Scotland is some 0.3 m higher than off Cornwall, at a distance of 800 km (Kelsey 1970), while figure 2 suggests that sea level *falls* to the north in the neighbouring ocean. The 0.6 m westerly rise across the U.S.A. is fairly well reproduced, but there are again serious discrepancies in the north–south levelling figures for both American coasts. Calculations for the shallow water between ocean and coast alter the picture in detail, but the larger discrepancies remain. Protagonists of both land and sea levelling have repeatedly searched their assumptions and methods for possible errors, but no systematic error of sufficient magnitude has yet been identified in either body of work.

The picture from ranged altimetry will be very different from the above. The gravitational level surface, which in a global context we may term 'the Geoid', is now known from gravity surveys and analysis of satellite orbits to have large undulations with amplitudes up to 100 m relative to the mean ellipsoid. Modern geoidal maps are too well known to be worth reproducing here; they are surveyed by King-Hele (1975). Most recently, the Skylab altimeter has revealed smaller scale features due to submarine trenches and seamounts. The pattern of sea-level anomalies, hopefully not too unlike figure 2, will appear superimposed on the geoidal structure and at first sight will be swamped by it, however accurate the measurement. Resolution of the true oceanographic anomalies in sea level will depend on accurate estimation and subtraction of the Geoid. This appears to be possible to about 1 m accuracy, but accuracy will improve as more details of the gravitational field become available. It is also possible to make direct measurement of the Geoid in local areas, using specialized shipborne equipment (Von Arx 1966). A confirmation of either land- or sea-based estimates of the anomalous slope in mean sea level in at least some areas seems possible from a combination of measurements.

Conversely, if a mean level map such as figure 2 can be confirmed in part, it can be used as a whole to convert the mean geocentric sea levels into a very accurate map of the Geoid.

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(*b*) *Ocean tides*

The principal *dynamic* feature of the ocean surface which may be sensed by altimetry is the vertical tide. Tidal characteristics at most coastal ports have been known for more than a century, but it is very difficult to deduce the form of the tidal waves in the deep ocean from their coastal values. A map which purports to show the amplitude and phase of one of the harmonic constituents of the tide at all points of the ocean is known as a 'cotidal map'. Such maps are not only of fundamental interest to tidal hydrodynamicists but are in growing demand for applications to studies of crustal loading (Farrell 1972), and of lunar variations in the atmosphere (Chapman & Lindzen 1970), and in the geomagnetic field (Malin 1973). However, despite great advances made in the last twenty years since the introduction of large computers, no completely satisfactory cotidal map for the world oceans has yet been produced.

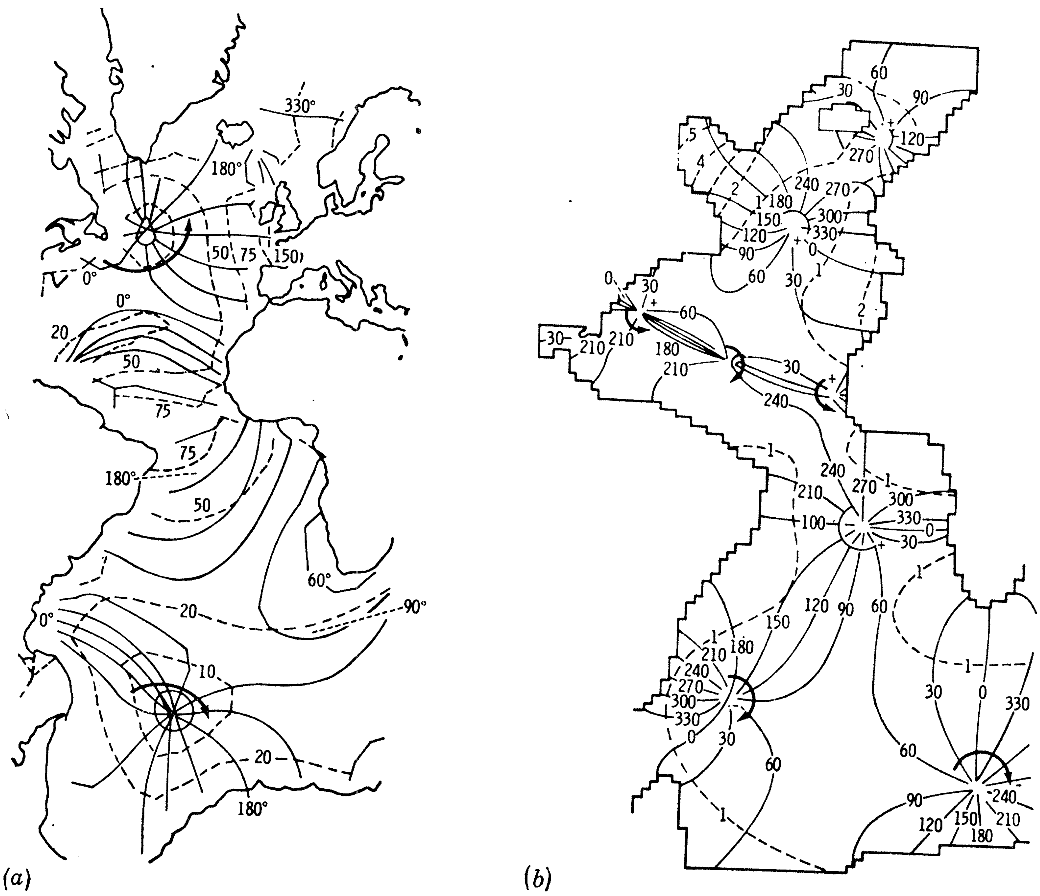

FIGURE 3. Two computed cotidal maps of the M_2 tide in the Atlantic Ocean. Left: by Hendershott (1972), using coastal data. Right: by Pekeris & Accad (1969), without coastal data. Both maps are computed for a rigid Earth.

Figure 3, extracted from Hendershott's (1973) comprehensive review, illustrates the current situation. It shows the results of two computations of the cotidal map for the M_2 constituent in the Atlantic Ocean. Other oceans were included as an integral part of the computations, but are not shown here. The map by Pekeris & Accad (right) used no empirical data, other than the topography of the ocean and the gravitational potential of the Moon. Hendershott's (left) map additionally used boundary conditions based on observed coastal data. The two maps appear to agree qualitatively in the North Atlantic, but in fact differ seriously in phase and amplitude. In the South Atlantic, they differ radically in their distributions of nodal points. There are similar differences in the other oceans and in comparison with the two or three cotidal maps computed by other authors. A definitive map seems curiously elusive. Hendershott (1972) showed that allowance for the elastic yielding of the Earth to the ocean tide would radically alter the solution, but he has so far been unable to solve the relevant equations completely.

The energy dissipated in the global tides is another longstanding problem, closely related to the increase in the length of the day and to the evolution of the lunar orbit. It may be estimated from the frictional stress on currents in all shallow seas, or from the work done by the gravitational forces integrated over the whole cotidal map. Lambeck (1975) has shown that all existing cotidal maps give roughly the same answer, about $3.7 \times 10^{12}\,\mathrm{J\,s^{-1}}$, despite their differences in detail. This shows that their lowest order harmonics are roughly equivalent. However, direct calculations of the frictional dissipation in shallow seas give only about $1.7 \times 10^{12}\,\mathrm{J\,s^{-1}}$. There is thus a major oceanographic problem (appreciated for some years) to find a physical sink for about half the tidal energy put into the oceans by the lunar and solar gravitational forces. Conversion into internal waves at shelf boundaries, and lateral eddy viscosity have been suggested, but neither has been convincingly proven. Establishment of a definitive cotidal map, correct in all details in even one part of the ocean, may help oceanographers to solve this problem.

One apparently obvious way to fix the oceanic cotidal maps is by direct measurement in the deep sea. Modern technology has made this possible only in the last decade by the development of high-precision bottom-pressure recorders. About six independent designs of deep-sea bottom-pressure recorder have been accomplished, mainly by scientific groups in the northern hemisphere (Anon. 1975). They have accounted for some 50 pelagic tidal stations other than islands. But these have so far made little impact on the global situation, for which an order of 10^3 such stations are necessary. Deployment of such recorders is slow, and expensive in ship time, while large areas of the southern hemisphere are seldom visited by oceanographic ships. It is ironic that one of the best-recorded sets of tidal data yet made in the deep ocean, from the MODE area south of Bermuda (Zetler *et al.* 1975), was found to agree better with a hand-drawn cotidal map from the pre-computer age than with any of the recently computed solutions as in figure 3.

The chief attraction of ranged altimetry as a means of determining oceanic tides is the potential ability to cover large areas of ocean which would be extremely slow and expensive to reach by ship-borne techniques. Unlike the static problem, there is no need to distinguish between geoidal and stationary oceanographic features, which would be regarded as a compound undulatory surface, independent of the time. The problem here will be to detect the variations in level of the same area of ocean surface on a large number of successive traverses. Since tidal amplitudes in mid-ocean are typically less than 0.5 m, this will demand the highest possible accuracy from the system, but tidal phenomena have the great advantage of being highly coherent with the calculable tidal potential of Moon and Sun, which makes them extractable from a relatively high noise background. Zetler & Maul (1971) have estimated that useful tidal analyses can be achieved with noise levels equal to the tidal amplitude, given a few hundred independent measurements in effectively the same site. Here, 'noise' includes meteorologically induced time dependent motions of the sea surface, but much of this can be removed in terms of known variations in atmospheric pressure. Other aspects of the tidal analysis of altimetry data will be discussed in the last section of this paper.

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A special feature of geocentric estimates of oceanic tides is that they include the tidal motion of the solid Earth – the 'Earth tide'. This is excluded from all ordinary marine tidal measurements, which are perforce made with instruments attached to land or to the sea bed. The Earth tide has amplitudes comparable with the ocean tide. It is usually separated into two parts, the 'body tide' of direct yielding to the tidal forces, amplitude about 0.3 m, easily calculable from theory; and the 'loading' tide due partly to the direct load of the oceanic tide and partly to self-gravitational attraction, amplitude of order 0.2 m. The loading tide is much more difficult to calculate from theory, since it depends on at least a good estimate of the global cotidal map. However, recent success achieved by various workers in calculating the loading tide on land, where it can be directly measured, suggests that there will be no difficulty in disentangling the Earth tide from the oceanic tide in measurements from ranged altimeters. Its presence gives added interest.

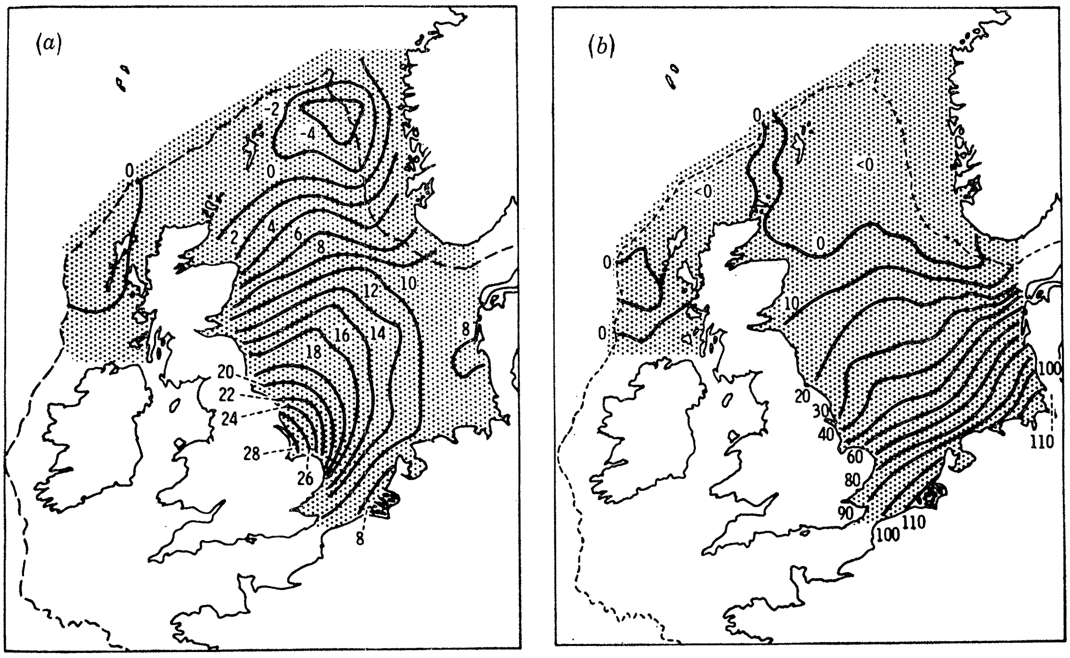

FIGURE 4. Contours of surge elevations (excluding the astronomical tide) in the North Sea, computed by Heaps (1969). Left: 16 h, 14 September 1962. Right: 00 h, 17 February 1962. Units are tenths of a foot (3 cm). The shaded area delimits the area of computation.

(c) *Storm surges*

The major disturbances in sea level due to extreme weather conditions, known as 'storm surges', are probably the only other dynamic feature easily recognizable in altimetric measurements. They occur principally in shallow water areas, because of the enhanced effect of wind stress there, and being transient phenomena confined to limited areas, they would not be very frequently observed by a given satellite altimeter. Moreover, laser ranging may be difficult under storm conditions. But any observations would be valuable.

Figure 4 shows contours of some storm surges in the North Sea, computed by Heaps (1969) by a finite-difference model. Elevations reach 3.5 m in one case (11 ft are indicated), causing severe flooding and loss of life and property. The ordinary tide has to be added to the surges shown, increasing the total sea-level anomaly to some 6 m in places. As with oceanic tides, storm surges are normally recorded only at the coastline where tide gauges are sited. There the models give fair agreement, and can be used to give reasonable forecasts of flood danger. But the coastline is an unsatisfactory place for testing models, because of local distortions to the wave form caused by shallow water, bays and headlands. Offshore observations would be better, but are normally difficult to obtain. The potential benefit of occasional measurements from ranged satellite altimeters is obvious.

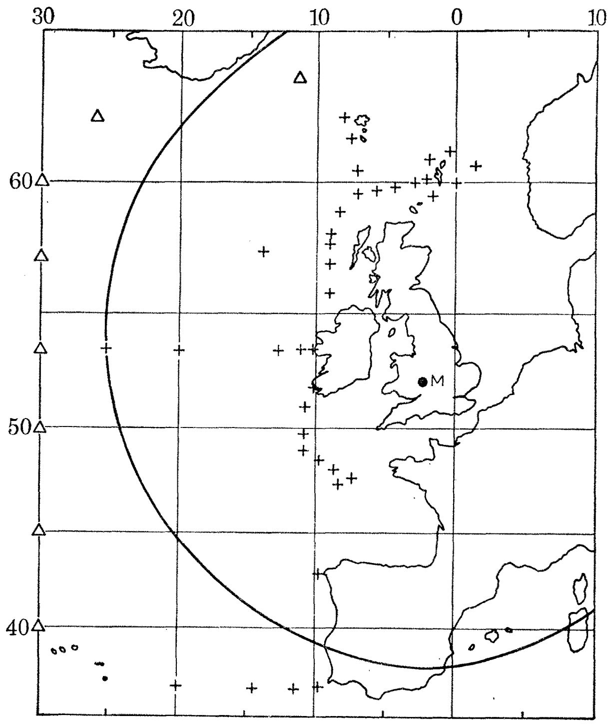

FIGURE 5. A possible oceanic observing area, based on a hypothetical laser tracking station in mid-Britain (M). The circle (distorted by the map projection) delimits a reasonable area for ranged altimetry. Crosses denote oceanic sites where tides have been measured to date. Triangles are similar sites for measurements in the near future.

POSSIBLE ANALYSIS PROCEDURES

To summarize the preceding paragraphs, a traverse of the ocean surface measured geocentrically by laser ranging to a satellite-borne altimeter will include contributions from:

(1) the Geoid (50 m),
(2) static anomalies in mean sea level (1 m),
(3) variable anomalies due to atmospheric pressure (0.2 m),
(4) oceanic plus Earth tides (0.8 m),
(5) unaccounted errors (0.5 m),

where the figures in parentheses are typical amplitudes. It would seem at first sight to be difficult to separate these elements, but several reasonable approaches are in fact possible, as already suggested. Since the variations of the Geoid are so much greater than any of the others, it is possible to treat all of 2–5 as random errors, and thus obtain a reasonable picture of the Geoid to about 1 m accuracy by a simple smoothing procedure, but this would not help oceanography. More usefully, subtraction of one of the latest models of the Geoid, known from gravimetric and satellite orbital data, would reduce the variations of (1) to something less than a metre, except in areas of unusual topography (seamounts, etc.), which could be avoided in subsequent analysis by correlating to known features of the sea bed.

It would then be convenient to group the variables into static (1, 2) and time-dependent (3–5). (3) can be practically removed by synoptic meteorology, and there is no need to discuss further the separation of (1) from (2) or of Earth and oceanic tides. The observed altitudes A can then be thought of in the form

$$A(\theta,\lambda,t) = S(\theta,\lambda) + \boldsymbol{T}(\theta,\lambda)\cdot\boldsymbol{G}(t) + R(t),$$

where θ, λ, t represent latitude, longitude and the time, respectively, S is the residual static anomaly (including oceanographic mean sea level), $\boldsymbol{G}$ is a multi-dimensional vector, calculable from the tide-generating potential, and $\boldsymbol{T}$ the corresponding oceanic response function to be evaluated (Munk & Cartwright 1966). R represents random noise, to be eliminated essentially by some averaging process. In practice, the object would be to evaluate S and $\boldsymbol{T}$ at a finite network of points, between which they are assumed to vary linearly in (θ, λ).

Direct evaluation of S, $\boldsymbol{T}$ by a least-squares fit to observed data is unlikely to be feasible until vast quantities of data have been accumulated. Rather, trial values of the complete function $\boldsymbol{T}$ would be supplied by variants of a tidal model, and the best function chosen as that which minimises the variance in time of $(A - \boldsymbol{T}\cdot\boldsymbol{G})$, averaged over all grid points. Some such scheme is planned by various American groups working on a multi-ranged area southeast of the United States (Leitao, Purdy & Brooks 1975).

Figure 5 shows a conceptual area of study for the seas surrounding Britain, assuming a laser ranging station in the Midlands (position unimportant). The advantages of working in this area, apart from a general extension of the American-based system, are:

(*a*) the existence of a dense network of directly measured pelagic tidal stations, which are to be used as boundary conditions to a sequence of models with various laws of dissipation;

(*b*) the large range of the tide itself in the area (of order 3 m);

(*c*) the relatively slight gradients of the Geoid, and relative absence of seamounts and other disturbing features to the gravitational field;

(*d*) the interest in the local marine geodesy relevant to the land-levelling network in Britain and western Europe;

(*e*) the ability to monitor storm surges in the North Sea.

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